

BACKGROUND OF THE INVENTION

1. Field of the Invention

This invention pertains in general to a system and method for the accurate estimation of the position of an underwater vehicle and more particularly to a system and method for the estimation and tracking of an underwater vehicle's position using a combination of bathymetry data, and the underwater vehicle's dynamics data in relation to a position marker.

2. Description of the Prior Art

Underwater vehicles are in common use for surveying and locating objects in the ocean due to their relatively low operation cost. To be useful however, the position of an underwater vehicle must be known so that the data collected by the towed vehicle's sensors can be georectified. Due to ocean dynamics, currents, waves, temperature fluctuation and etc., acting on underwater vehicle's tether or on the vehicle itself, a submerged vehicle's position relative to the towing vehicle or some other georectified point is often difficult to determine. Since the ocean is opaque to high-frequency electromagnetic signals, the global positioning system (GPS) generally cannot be used for submerged vehicle positioning.

Current methods for determining the position of an underwater vehicle include the lay-back model (See, FIG. 1), the long base line (LBL) model (See FIG. 2), the short base line (SLB) model (See FIG. 3), the inverted SLB system and the Localization System (LOST) (See FIG. 4). These systems are generally configured to manage the position of a tethered or towed vehicle and are ill equipped for a remote or untethered underwater vehicle.

With the lay-back model method the underwater vehicle is tethered and is assumed to be directly behind the towing vehicle and the lay-back (distance from the towing vehicle) is assumed to be a fixed multiple of the tow vehicle's depth. Alternatively, the amount of cable paid out is assumed to be the slant range to the tow vehicle. The inverted SLB method does not take into account the cable's cantilever. The neither the lay-back, the LBL, or the SLB methods take into account the effects of local current on the cables' shape and position. The accuracy of these methods will in general suffer as a function of time and therefore not support precise positioning of the towed vehicle.

The LBL method uses a series of acoustic transponders to localize the tow-vehicle by measuring the time delays between the tow-vehicle and the transponders. These transponders may be bottom mounted or localized on the surface, such as shown in U.S. Patent 5,119,341. While these systems are very accurate, their deployment is a time consuming and expensive operation. Furthermore the LBL method requires that the systems be redeployed to each operational area.

The SBL method uses time delay and measured arrival angle of an acoustic signal from the tow-vehicle to the towing vehicle to compute position. These systems are fairly accurate but their resolution is range dependent. Typically, in order to achieve sufficient accuracy, a SBL system is deployed over the tow-vehicle, requiring a second surface craft-an obvious economic disadvantage.

With the inverted SBL method, the directional acoustic receiver is mounted on the towed vehicle instead of the towing vehicle. As a consequence the range dependent accuracy of this system is a problem for very deep water operation. SBL and inverted SBL systems cannot be readily used in applications other than ship towed systems.

The LOST 1, localization method, the position of a towed underwater vehicle is generated via the use of bathymetry data collected by the towing vessel and the depth range of the underwater vehicle from the towing vessel as shown in U.S. Patent 6,256,264 incorporated herein by reference. LOST 1 uses a single dimensional model to generate the underwater vehicle's position assuming the underwater vehicle's position along track doesn't change. This feature limits the deployment of the underwater vehicle to an area directly behind the tow vessel in order to achieve sufficient accuracy.

Summary

A system for the accurate determination of the position of an underwater vehicle comprising a system observer subsystem having a state velocity update module, a terrain matching module, means for generating a prediction of the terrain matching module's performance and a constrained extended Kalman filter subsystem. The constrained extended Kalman filter subsystem includes a steady state extended Kalman filter, a non-linear constraint module, and a state predictor. The system observer integrates bathymetry data corresponding to the area of the submersible vehicle, with the vessel's measured ocean depth, the vessel's predicted state, the vessel's measured velocity into a terrain based state estimate, a final predicted state, the Kalman filter takes the terrain based state estimate, the final predicted state, the measured slant range and the location of the known point and computes the final estimate of the vessel's position and a prediction of the vessel's position at the next time step. A method for the accurate determination of the position of at least one underwater vehicle comprising the steps of (1) acoustically coupling at least one underwater vehicle to a sea borne position marker having a known position; (2) predicting the at least one underwater vehicle's position, based on a past estimate of the underwater vehicle's position, and an estimate of its velocity over the sea bottom; (3) estimating the underwater vehicle's position utilizing measured ocean depth at the underwater vehicle's position, bathymetry data and the underwater vehicle's predicted position in a single point terrain match; (4) computing an estimate of the underwater vehicle's position based on the prediction of the at least one underwater vehicle's position based on vehicle dynamics and the estimated underwater vehicle's position based on ocean depth and bathymetry data; and (5) computing a corrected estimate of the at least one submersible vehicle's position that utilizes the estimate of the

underwater vehicle's position and a measured slant range from the at least one submersible vehicle to the sea borne position marker whose position is known.

BRIEF DESCRIPTION OF THE DRAWINGS

Figure 1 shows a cable lay back method of the prior art.

Figure 2 shows a long base line (LBL) approach of the prior art.

Figure 3 shows a short base line (SBL) approach of the prior art.

Figure 4 shows a localization system (LOST 1) approach of the prior art.

Figure 5 shows an overview of the implementation of the LOST 2 system employing a floating buoy position marker.

Figure 6 shows a top view of the LOST 2 system employing multiple underwater vehicles.

Figure 7 shows an example implementation of the LOST 2 system employing a position marker anchored on the sea floor.

Figure 8 shows a block diagram of the LOST 2 system.

Figure 9 shows a block diagram of the system observer subsystem.

Figure 10 shows a block diagram of the signal flow of the state velocity update module.

Figure 11 shows a block diagram of the signal flow of the terrain matching module.

Figure 12 shows a block diagram of the signal flow for the spatial based performance prediction module.

Figure 13 shows a block diagram of constrained extended Kalman filter sub-system.

Figure 14 shows a block diagram of the signal flow for the steady state Kalman filter.

Figure 15 shows a block diagram of the signal flow for nonlinear constraint module.

Figure 16 shows a block diagram of the signal flow for the state predictor module.

Detailed Description

The LOST-2 underwater positioning system is a system for the accurate determination of the position of an underwater vehicle. The system and method combines parts of dead-reckoning, acoustic-based, and terrain-based positioning into a single integrated system and apply these various methods in an underwater environment. The system is designed in two basic subsystems, the system observer and the constrained extended Kalman filter. Prior to discussing the figures a brief description of the basic architecture of each subsystem follows, however the basic grouping of the operative systems and sub systems are not limited to the descriptions below.

The LOST-2 system uses a single position marker located at some known point. The position marker is acoustically coupled to one or more submersible vehicles. This allows the underwater vehicle's range from the position marker to be computed using an acoustic ranging device. Bathymetry data and dynamic data are also employed in concert with the range data to estimate and predict the underwater vehicle's position.

The LOST-2 employs a system observer subsystem having a state velocity update module, a terrain matching module and means for generating a prediction of the underwater vehicles spatial performance. The second subsystem, the Kalman filter

subsystem comprises a constrained extended Kalman filter subsystem having a steady state extended Kalman filter, a non-linear constraint module, and a state predictor.

The system observer integrates bathymetry data corresponding to the area of the submersible vehicle with the vessel's measured ocean depth, the vessel's predicted state, the vessel's measured velocity, and the measured slant range and location of a known point into a terrain based state estimate, a final predicted state and a measured slant range to the known point. This information is used by the second subsystem, the Kalman filter to generate a final estimate of the underwater vehicles position.

Specifically, the Kalman filter takes the terrain based state estimate, the final predicted state, the measured slant range and the location of the known point and computes the final estimate of state and a prediction of the next system state to generate an estimate of the position of the underwater vessel.

Referring now to the figures, Fig. 5 shows a general overview of the implementation of an example embodiment of the LOST 2 system using a single transponder equipped buoy as the position marker. In applications where high resolution bathymetry data is available, a technique exist for the measurement of the ocean depth 590 at the position of the underwater vehicle and a technique exist for the measurement of the slant range 560 of the underwater vehicle to a position marker 520. The position marker must be easily related to a known point. A single point terrain match based on bathymetry data 550 for the local area is also used for underwater vehicle localization. LOST 2 also uses the underwater vehicle's velocity data as well as other underwater vehicle dynamic information to estimate and predict the underwater vehicles position. In this example embodiment the underwater vehicle used is untethered, a feature allowed by the use of 2 dimensional modeling discussed below. Briefly, the LOST 2 system employs a 2 dimensional model addressing the underwater vehicle's longitudinal track and the underwater vehicle's cross track. This feature allows LOST 2 to generate accurate position data when both the longitudinal track as well as the cross track of the underwater vehicle vary. Via the use of 2 dimensional modeling (longitudinal track and cross track) and a single position marker, the LOST 2 system may deploy and track the location of several untethered underwater vehicles simultaneously.

The LOST 2 system requires several input parameters. These include the (1) geo-rectified, high resolution bathymetry 550 for the operational area collected prior to the underwater vehicles deployment , (2) the depth 530 of the underwater vehicle 510 through the use of a pressure sensor or with an up looking acoustic ranging device, (3) the velocity of the underwater vessel 570 both course and speed (4) and the slant range 560 to the position marker 520 via a acoustic ranging device mounted on the position marker or on the underwater vehicle itself. The position marker is located a known point which is preferably a geo-rectified 580 point. The LOST 2 system's output is the estimated geo-rectified position of the underwater vehicle 510.

Figure 6 shows the top view of an example embodiment a LOST 2 system operating and providing the position of 4 untethered underwater vehicles simultaneously 600. A single

position marker 620 having a known position is acoustically coupled to untethered underwater vehicles 610, 610', 610'', and 610'''. Preferably each underwater vehicle 610 is equipped with a simple transponder which is acoustically coupled to position marker 620 which may be a floating buoy or located on the sea floor. By acoustic coupling to the position marker, the slant range of each underwater vehicle to the single known point 670, is measured. For the area in which each of the underwater vehicles operates there must exist geo-rectified, high resolution bathymetry data collected prior to the underwater vehicles deployment. When operating multiple underwater vehicles LOST 2 requires a depth measurement for each underwater vehicle 601 and the velocity of each of the underwater vessel's including both course and speed.

Independently, for each underwater vehicle LOST-2 predicts the underwater vehicle's position, based on a past estimate of the underwater vehicle's position, and an estimate of its velocity over the sea bottom. LOST 2 then estimates the underwater vehicle's position utilizing measured ocean depth at the underwater vehicle's position, bathymetry data and the underwater vehicle's predicted position in a single point terrain match. The system then computes an estimate of the underwater vehicle's position based on the prediction of the underwater vehicle's position based on vehicle dynamics and the estimated underwater vehicles position based on ocean depth and bathymetry data. LOST 2 then computes a corrected estimate of the submersible vehicle's position that utilizing the estimate of the underwater vehicle's position and a measured slant range from the underwater vehicle to the sea borne position marker whose position is known. These steps are repeated for each underwater vehicle using bathymetry data and dynamic information relevant to that particular underwater vehicle.

Figure 7 shows yet another example embodiment of the LOST 2. This embodiment of the LOST 2 system also uses a single position marker 720 located on the sea floor acoustically coupled to underwater vehicle 710. Similar to the example embodiment shown in figure 5 the position marker is located at a known position, preferably the location of the position marker is geo-rectified.

Figure 8 shows a block diagram of the algorithm employed by the LOST 2 system. The LOST 2 system comprises two basic subsystems, the system observer 810 and the constrained extended Kalman filter 820. These two subsystems are composed of six modules. The basic grouping of the operative modules and sub systems are not limited to the groupings as described above or below.

The system observer subsystem is composed of three modules: a state velocity update 850, terrain matching module 840, and means for generating a prediction of the terrain mapping performance shown in this example as spatial performance prediction module 830.

The constrained extended Kalman filter subsystem 820 consists of three modules: a steady state extended Kalman filter 870, a non-linear constraint module 880, and a state predictor 890.

System observer 810 takes in all the available measurements and generates two position estimates, one from Newtonian motion and the other based on the terrain in the area. As shown in Figure 8, system observer 810 integrates the high resolution bathymetry 811, vessel's measured ocean depth 812, vessel's predicted state 13, the vessel's measured velocity 814, and the measured slant range and location of the known point 815 into the terrain based state estimate 841. The system outputs the terrain based state estimate 841, a final predicted state 860 and the measured slant range and location of the known point. The system observer also computes a prediction of the performance of the terrain mapping system shown in figure 8 as a spatial performance estimate 831.

The constrained extended Kalman filter 820 takes the terrain based state estimate 841, a final predicted state 860, the measured slant range and location of the known point 815 and computes the final estimate of state 899 and a prediction of the system state at the next time step 898.

The inputs the system requires are a detailed, high resolution bathymetric map in the operational area 811, a slant range to a known position 815, an estimate of the underwater vehicle's state 813, a measured or estimated velocity 814 and the depth of the ocean at the position of the underwater vehicle 812.

Referring again to the embodiment shown in figure 5, the high resolution bathymetric map in the operational area is shown as 550, the slant range to a known position 560 is effective the range from underwater vehicle 510 to position marker 520, an estimate of the underwater vehicle's state which is generated by the LOST 2 system (not shown), a measured or estimated velocity of the underwater vehicle 570 and the depth of the ocean 590 at the position of underwater vehicle 510. These inputs are used in conjunction with the output of the other modules to generate an accurate estimate of the position of the underwater vessel 510.

Figure 9 shows a block diagram of the System Observer subsystem 910. As stated earlier, this subsystem's primary function is to generate all the predictions of the state that are required by the constrained extended Kalman filter. It also provides a prediction of performance based on the spatial distribution of the bathymetry. This subsystem integrates the high resolution bathymetry, vessel's measured ocean depth, vessel's predicted state, the vessel's measured velocity, and the measured slant range and location of the known point into the terrain based state estimate. The subsystem's outputs are the terrain based state estimate, the final predicted state and the measured slant range and location of the known point.

The first subsystem component is the state velocity update, whose signal flow diagram is shown in figure 10. The state velocity update module 850 serves to provide an estimated location of the underwater body. It does this by combining the best available velocity 814 (speed and heading), and the best estimate of state 830 to produce the best estimate state prior to the terrain match. This module then outputs the predicted state 840 at the present time.

The output of the state velocity update 840 is sent to the terrain matching module, whose signal flow is shown as figure 11. The terrain matching module is the part of the system that serves as the independent system observation. The observation is preferably made by comparing the measured ocean depth 1112 to the bathymetry 1111 in the operational area. This accomplished by means of a maximum likelihood estimator. The output of this module is the terrain based state estimate 1141, which is also the independent system observation.

The final module of the system observer is the spatial based performance prediction. The signal flow is contained in figure 12. The spatial performance prediction module 1230 produces an estimate of how well the system is capable of performing in a given area 1231 based on the high resolution bathymetry 1211 in the area of the underwater vehicle.

The system observer generates two separate estimates of the position of the underwater body. The first position estimate is generated in the velocity update. This position estimate is then used in conjunction with bathymetry to generate the terrain based position estimate.

State Velocity Update module

Figure 10 shows a block diagram of the signal flow of the state velocity update module. The state velocity update module receives the vessel's predicted state using all of the information before the current time step 1013, and the vessel's measured velocity 1014 and computes the vessel's final predicted state 1060. The vessel's final predicted state is the underwater vehicle's position, $P_p(n)$, based on a past estimate of the underwater vehicle's position, $P_e(n-1)$ and an estimate of the underwater vehicle's velocity over the sea bottom.

The velocity update module serves as the dead-reckoning system and computes an estimated position for the vessel in the along track, cross track coordinate system based on a measured velocity vector for the underwater vessel, and the best prediction of position of the underwater vessel from the previous time step. The velocity update calculates the new position (along track, cross track) as two separate and independent processes, one in each direction. This is done throughout the system.

For each direction, the position estimate is computed by adding the measured velocity, multiplied by time as appropriate, to the previous position estimate.

The formulas used to compute each part of the estimate are:

$$\hat{x}_n^a = x_{n|n-1}^a + v_n^a \quad (3.3)$$

$$\hat{x}_n^c = x_{n|n-1}^c + v_n^c \quad (3.4)$$

where the superscripts a and c correspond to along track and cross track and v is the measured velocity. The subscript $n|n-1$ refers to the estimate of the state (x) at time (n) given all information available at time $n-1$. Therefore, $x_{n|n-1}$ is the best estimate of the position before the time n measurements, v_n is the velocity times the time between samples at time n . The resulting prediction of position is provided to both the terrain matching module, and the constrained extended Kalman filter. This prediction is used by the terrain matching module to generate the second observation of position.

Terrain based position estimate

The terrain matching module receives the bathymetry data, a measurement of the ocean's depth at the vessel's position and the vessel's final predicted state and computes the terrain based state estimate with a single point position match.

The terrain matching module takes as its inputs the position generated by the velocity update module, the measured ocean depth at the vessel's position and the high resolution bathymetry for the area of operation. The module then generates the best estimate of position based on maximum estimation principles.

In the preferred embodiment, a reasonably dense, spatially varying bathymetric data set in the neighborhood of the vessel is used. Each point in this bathymetric data set is a triple, $a_i = (x_i, y_i, z_i)$. Where x_i, y_i is the 2-dimensional position vector (eastings and northings) in meters and z_i is the measured depth in meters. In a preferred embodiment this data has been previously collected however it may be collected concurrently with the underwater vehicle deployment.

Given the dead reckoning estimate of vessel position χ_n and the measured ocean depth at the vessel's position, z_n , the 3 dimension position vector, $a_n = (\chi_n, z_n)$, can be formed using the same units as a_i after rotating coordinate systems. Without loss of generality, it is assumed that the ocean bottom depth is measured directly beneath the vessel to simplify this exposition; in practice it may be necessary to correct for vessel orientation, which is possible through a set of rotations. If the true location of the vessel and ocean bottom depth is denoted as $a_n = (x, y, z)$, then the measurement error, defined as the difference between the true and dead reckoned positions, can be written as $e = a_n - a_{n|n-1}$.

Viewed as an estimation problem, a maximum likelihood approach is used. That is, given several observations from a distribution with unknown parameters the value of the observation that maximizes the likelihood function is the best estimate of the parameters. The a_n and $a_{n|n-1}$ are related through the likelihood

function (LF) based on a known or assumed error distribution, f_e .

The LF can be expressed as:

$$L(\alpha_i | \alpha_n^-) = f_e(\alpha_i - \alpha_n^-) \quad (3.5)$$

Given the estimated position α_n^- and the bathymetry in the area $\{a_i\}$ the most likely location of the vessel constrained to be in the bathymetric data set can be determined as that a_i which maximizes $L(a_i | \alpha_n^-)$. When applied to the bathymetric data set $\{a_i\}$, (3.5) produces a measure which can be viewed as the weighted distance from the estimated position, \hat{x}_n . Selecting the bathymetry triple which minimizes this distance is the maximum likelihood estimate of position constrained to the bathymetric data set.

The Terrain matching module employs the following relation:

$$\lambda(\alpha_i | \alpha_n^-) = (\alpha_i - \alpha_n^-)^T \Sigma^{-1} (\alpha_i - \alpha_n^-) \quad (3.7)$$

where α_i (alpha i) refers to data points from the bathymetry set, α_n^- (alpha n) is the 3D position vector formed by the measured depth at the position of the underwater vessel and the estimate of vessel position. (from the state velocity update) One assumes the error e is to be zero mean Gaussian with covariance Σ .

Because Σ is a covariance matrix and it is positive semidefinite, the likelihood function in (3.7) is strictly non-negative with minimum 0 at $a_i = \alpha_n^-$. The correct value for the covariance matrix varies depending on the nature of the bathymetric data being used.

The matrix given by Eqn. 3.8 contains the assumed variances in the position for each of the 3 axes.

$$\Sigma = \begin{bmatrix} \sigma_a^2 & 0 & 0 \\ 0 & \sigma_c^2 & 0 \\ 0 & 0 & \sigma_z^2 \end{bmatrix} \quad (3.8)$$

where σ_a is the variance in the along track direction, σ_c is the variance in the cross track direction and σ_z is the variance in the z direction. The resulting position from the Terrain matching module is the α_i (alpha i) that minimized Eqn. 3.7. Substituting this matrix into Eq. 3.7 and performing minimization generates the MLE's observation of position which is then passed to the constrained Kalman filter.

Three of the modules in the LOST2 system form the second subsystem of the LOST2 system, a constrained extended Kalman filter. Figure 13 shows a block diagram of the constrained extended Kalman filter 1320 subsystem. The Kalman filter subsystem comprises a steady state extended Kalman filter 1370, a non-linear constraint module 1380, and a state predictor 1390. The steady state extended Kalman filter 1370 features nonlinear constraints that are placed on it to improve performance and stability given the poor nature of all models for underwater vessels.

The signal flow diagram of the constrained Kalman filter is shown in figure 13, builds on Kalman filter principles, extended to nonlinear systems, to integrate all of the state estimates into a single final state estimate 1399. This subsystem takes as input the terrain based state estimate 1341, a final predicted state 1360, the measured slant range and location of the known point 1315 and computes the final estimate of state 1399 and a prediction of the next system state 1398.

The extended Kalman filter that takes the previous estimate of the underwater vehicle's position, $Pe(t-1)$, and the maximum likelihood estimator's estimate of the underwater vehicle's position, $MLE(t)$, and computes a linear Kalman filter position estimate at time (t) , $KPAT(t)$.

The steady state Kalman filter, contained in figure 14, is the heart of the constrained extended Kalman filter subsystem 1320. Referring now to figure 14, this filter merges the two estimates of state in accordance with Kalman filter theory. This module takes as input the two estimates of state, the vessel's final predicted state, 1460 and the terrain based state estimate 1441, and outputs a linear combination of the two 1475.

The steady state Kalman filter receives the vessel's final predicted state and a terrain based state estimate from said system observer subsystem and computes the linear filter's state estimate therefrom.

After the prediction and observation steps of the Kalman filter are completed, the position estimates of the system observer must be merged. The terrain based observation (y) is merged with the prediction of the velocity update by means of a linear filter. The linear extended Kalman filter is the merging step of the constrained extended Kalman filter.

This module works in two steps. First the error or innovations of the system are computed by subtracting the observation from the prediction. The second step is to merge the innovations with the prediction via Kalman filter theory.

The innovations are computed by differencing the position obtained from the terrain match module (y) from the estimate of position at time n given all information at time $n-1$. In essence the innovations define the new information that is available at the present time step (n). The innovations are computed in each of the two independent directions separately. These 'innovations' can be expressed as:

$$\eta_n^a = y_n^a - \hat{x}_{n|n-1}^a \quad (4.15)$$

$$\eta_n^c = y_n^c - \hat{x}_{n|n-1}^c \quad (4.16)$$

After the innovations are defined, they are merged with the prediction in each direction by the Kalman gain K . This can be done in this manner due to the assumption that the system decouples into two independent systems.

The best Kalman filter estimate of position in eqn 4.17 and 4.18 is obtained by adding a portion of the innovations to the estimate of position at time n given all information at time n-1. The 'portion' is determined by K , the Kalman filter gain which is heuristically set for this system (extended Kalman filter) vs. computed (standard Kalman filter). Again following the Kalman principles, this is expressed as:

$$\hat{x}_{n|n}^a = \hat{x}_{n|n-1}^a + K_n^a \eta_n^a \quad (4.17)$$

$$\hat{x}_{n|n}^c = \hat{x}_{n|n-1}^c + K_n^c \eta_n^c \quad (4.18)$$

This module would in a normal extended Kalman filter complete one cycle through the filter algorithm. However, due to the unique nature of underwater vessels and this system, nonlinear constraints have been added to improve the system's performance. Due to the errors in linearization and the lack of a good system model, the linear filter's output must be constrained. This is done in two stages. First, the amount of change from the predicted state is thresholded. Secondly, the slant range is used to force the estimated location to be the correct distance from the known point. The nonlinear constraints module forms the constrained part of the constrained extended Kalman filter. A signal flow diagram for the nonlinear constraints module is shown in figure 15.

The non-linear constraint module 1580 receives the underwater vehicle's measured slant range and location of the known point 1515, and the vessel's final predicted state 1560 from the system observer subsystem, and the linear filter's state estimate 1575 and generates an estimate the vessel's final state 1599.

The Kalman filter has two problems associated with it, each of which are addressed by a separate set of nonlinear constraints. The first set of constraints serves to cull outliers from the system by means of a threshold. The threshold limits the distance that the system is allowed to change the predicted location. The second constraint is a slant range from a known point. This serves to aid in convergence and to prevent divergence.

The first set of constraints serves to cull the outliers from the observation module. These outliers may cause unpredictable performance. This is accomplished

by comparing the difference between $\chi_{n|n-1}$ and $\chi_{n|n}$ in each direction and by limiting its magnitude to a certain value. Basically, it limits the change in estimated position from the Kalman filter in each axis to ϵ to prevent system divergence. This may be expressed as:

$$\hat{x}_{n|n}^a = \begin{cases} \hat{x}_{n|n-1}^a - \epsilon & \hat{x}_{n|n}^a - \hat{x}_{n|n-1}^a < -\epsilon \\ \hat{x}_{n|n}^a & \text{when} \\ \hat{x}_{n|n-1}^a + \epsilon & \hat{x}_{n|n}^a - \hat{x}_{n|n-1}^a > \epsilon \end{cases} \quad (4.19)$$

$$\hat{x}_{n|n}^c = \begin{cases} \hat{x}_{n|n-1}^c - \epsilon & \hat{x}_{n|n}^c - \hat{x}_{n|n-1}^c < -\epsilon \\ \hat{x}_{n|n}^c & \text{when} \\ \hat{x}_{n|n-1}^c + \epsilon & \hat{x}_{n|n}^c - \hat{x}_{n|n-1}^c > \epsilon \end{cases} \quad (4.20)$$

These limits have the effect of trading convergence time for a vast increase in stability. The unit ϵ is meters and a rule of thumb for a nominal value is 0.1 meters per second. This is scaled by the sampling rate.

The other set of system constraints is the slant range correction. This corrects the estimated location of the vessel by forcing it to have the correct horizontal range (magnitude only) from the transponder. It does this by forming a line starting from the transponder and ending at the estimated vessel position. This line is then scaled by moving the estimated position along the line until the range is correct. This serves to increase convergence time by allowing for large movements in position and prevents divergence by preventing positions that are not realistic given the slant range information.

The nonlinear constraints distinguish this filter from other Kalman filters and are an extension of extended Kalman filters dictated by the nature of the system. By introducing nonlinearities into the linear system model, the system predicts the real dynamics better. The output of this module completes the merging of new information into the predicted location and is the best estimate as to the vessel's position.

The final module in the constrained extended Kalman filter is the state predictor, which is shown in figure 16. The state predictor module serves as part of the prediction stage of the Kalman filter.

This module computes an estimated position for the vessel in the along track, cross track coordinate system 1698 based on previous position estimates (stored internally).

The state predictor 1690 receives the vessel's final estimated state 1699 from the non linear constraint module and generates a prediction of the vessel's state at the next time step 1698 using the data generated for the current time step.

The state predictor thus acts as a range corrector that utilizes the linear Kalman filter position estimate at time (t), KPAT(t), a sea borne position marker, and a measured slant range from the at least one submersible vehicle to the sea borne position marker and computes a final estimate of the at least one submersible vehicle's position.

This module projects the state ahead by keeping an internal running estimate of the unmeasured velocities and adds this to the final estimated state. This is the first stage in preparing one of the state estimates for the next time step.

The state predictor calculates the new position (along track, cross track) as two separate and independent processes, one in each direction. This is done throughout this system. For each direction, the position estimate is computed by adding a portion of the unmeasured estimated velocity in each direction. This is done by differencing the previous two position estimates and multiplying the result by $1-\mu$. The value of μ is different in each direction and for each specific system, but a ballpark figure for the correct value is 0.1.

This provides an estimate of the vessels position at the next time step given all information known now, and it also serves to incorporate any unmeasured velocity which can be expressed for each axis as:

$$\hat{x}_{n+1|n}^a = x_n^a + (1 - \mu_1)(x_n^a - x_{n-1}^a) \quad (4.21)$$

$$\hat{x}_{n+1|n}^c = x_n^c + (1 - \mu_2)(x_n^c - x_{n-1}^c) \quad (4.22)$$

Note that x at $n+1|n$ at time step n is equivalent to x at $n|n-1$ at the next time step which is the value used by the state velocity update module (with the measured velocity to determine the estimate of vessel position based upon dead-reckoning alone).

Note that the difference between x at n and $n-1$ is effectively using the last two position estimates to generate an estimate of vessel velocity independent of the vessel velocity input used by the state velocity updater. The resulting position estimate is then fed forward as one of the required inputs of the system observer.

Bathymetry data, position and velocity of the underwater vehicle over the sea bottom, ocean depth at the position of the underwater vehicle, the underwater vehicle's slant range to the position marker and the georectified location of the position marker may be stored in the memory or memory devices of a computer. The computer is also utilized to analytically determine the underwater vehicle's position using the bathymetry data, the slant range of the underwater vehicle to the position marker, position and velocity of the

underwater vehicle over the sea bottom as well as displaying and storing the computed underwater vehicle's position. This analytically determined information may be displayed on either a video monitor or a printing device.

Although this invention has been described in relation to the exemplary embodiment's thereof, it is well understood by those skilled in the art that other variations and modifications can be affected on the preferred embodiment without departing from the scope and the spirit of the invention as set forth in the claims: